7.3) Definite Integrals

Evaluation Theorem

If $f(x)$ is continuous on the interval [a,b] then:

$$\int_a^b f(x) \quad dx = F(b) - F(a)$$

where F(x) the antiderivative of f(x)

- *(This theorem is actually the 2nd part of the Fundamental Theorem of Calculus (FTC)).*

Example: Find $\int_0^2 (x^2+1) \quad dx$

$$= \left[\frac{x^3}{3} + x + c\right]_0^2 = \underbrace{\left(\frac{8}{3} + 2 + \cancel{c}\right)}_{F(2)} - \underbrace{\left(\frac{0}{3} + 0 + \cancel{c}\right)}_{F(0)} = \frac{14}{3}$$

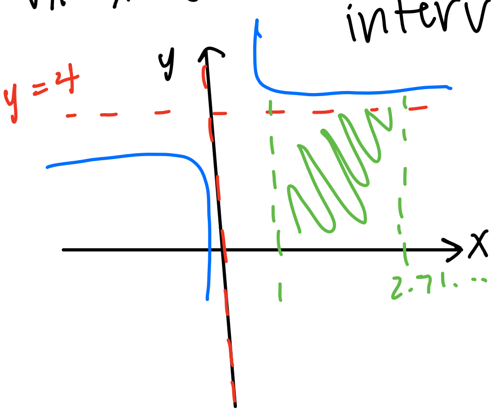
Example: Explain why the following function is integrable on the interval shown. Then compute the definite integral.

VA: x = 0 but not in the interval from 1 to e

$$\int_1^e \frac{1}{x} + 4 \quad dx = \left[\ln x + 4x\right]_1^e$$

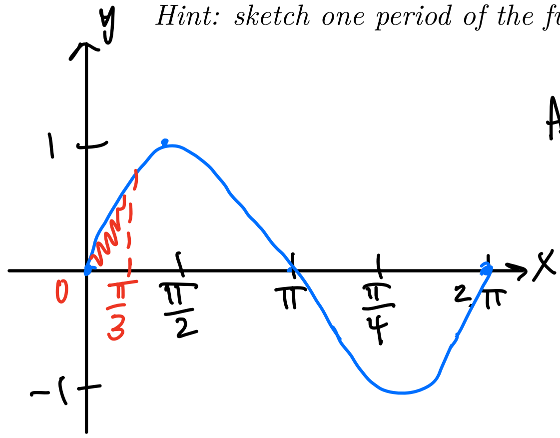
$$= (\ln e \cdot 4e) - (\cancel{\ln 1}^{\,0} + 4(1))$$

$$= 1 + 4e - 4 = 4e - 3$$



Example: Find the area enclosed by the x-axis, the curve $y = \sin x$ and the lines $x = 0$ and $x = \pi/3$.

Hint: sketch one period of the function first.



$$A = \int_0^{\frac{\pi}{3}} \sin x \, dx = \left[-\cos x\right]_0^{\frac{\pi}{3}}$$

$$= \left(-\cos\frac{\pi}{3}\right) - \left(-\cos 0\right)$$

$$= -\frac{1}{2} + 1 = \frac{1}{2}$$

Properties of definite integrals

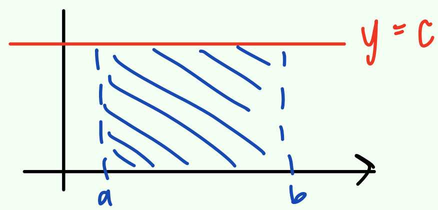
1. $\int_a^b f(x) \quad dx = -\int_b^a f(x)$

negate to flip a&b

"proof"

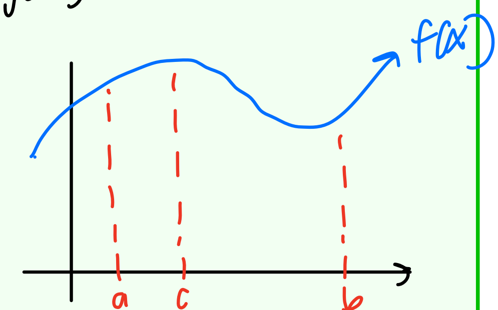
$-(F(a)-F(b)) = F(b)-F(a)$

2. $\int_a^a f(x) \quad dx = 0$ (zero area)



3. $\int_a^b c \quad dx = c(b-a)$

4. $\int_a^b [f(x) \pm g(x)] \quad dx = \int_a^b f(x)dx \pm \int_a^b g(x)dx$



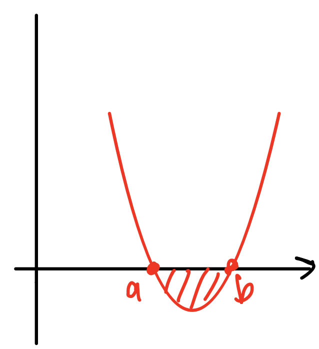
5. For a integrable function $f(x)$ and $a < c < b$

$$\int_a^c f(x) \quad dx + \int_c^b f(x) \quad dx = \int_a^b f(x)dx$$

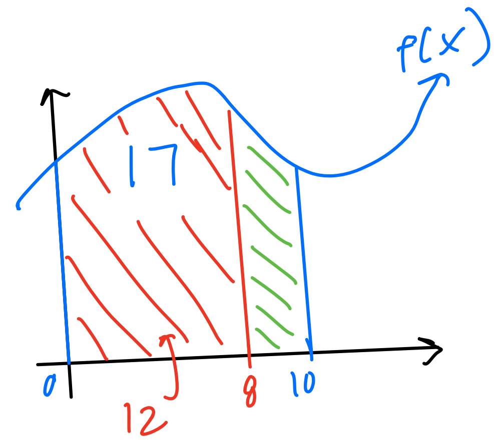
combine

- When dealing with areas we have to take care with regions below the x-axis...

$\Rightarrow$ definite integral will output a negative number



Example: If $\int_0^{10} f(x) \quad dx = 17$ and $\int_0^{8} f(x) \quad dx = 12$ find the following:



a) $\int_8^{10} f(x) \quad dx = 17 - 12 = \underline{5}$

b) $\int_3^{3} f(x) \quad dx = \underline{0}$

c) $-\int_{10}^{0} f(x) \quad dx = \int_0^{10} f(x)\,dx = \underline{17}$

d) $\int_0^{10} f(x) + 3x - 1 \quad dx = \underbrace{\int_0^{10} f(x)\,dx}_{} + \int_0^{10} 3x - 1 \, dx$

$= 17 + \left[\frac{3x^2}{2} - x\right]_0^{10}$

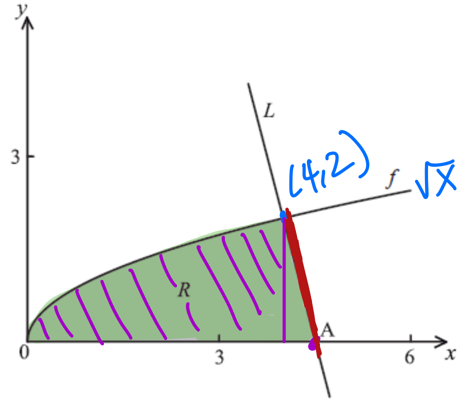
$= 17 + (150 - 10) - (0)$

$= 157$

Example: Use the properties of integrals to evaluate $\int_0^1 (4 + 3x^2) \quad dx$

Example: (SL Math DP Exam 2009)
Let $f(x) = \sqrt{x}$. Line L is the normal to the graph of f at the point (4, 2) .

In the diagram below, the shaded region R is bounded by the x-axis, the graph of f and the line L .



a) $f'(x) = \frac{1}{2\sqrt{x}}$

$f'(4) = \frac{1}{2\sqrt{4}} = \frac{1}{4}$ $m = -4$

$y - 2 = -4(x-4)$

$y - 2 = -4x + 16$

$y = -4x + 18$

a) [4 marks] Show that the equation of L is $y = -4x + 18$.

b) [2 marks] Point A is the x-intercept of L . Find the x-coordinate of A.

c) [3 marks] Find an expression for the area of R .

b) $0 = -4x + 18$

$-18 = -4x$

$x = \frac{9}{2}, 4.5$

c) $R = \int_0^4 \sqrt{x}\,dx + \int_4^{4.5} -4x + 18\,dx$

↑answer! bc it asks for expression not total answer/area