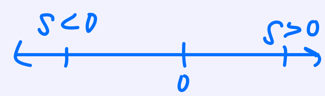
8.3) Kinematic Motion

Kinematics: the study of movement

time t: independent variable. $t=0$ to be a convenient time for w

Displacement vs. Distance traveled

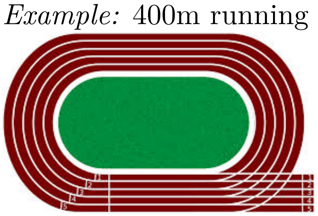
Displacement s: how far we are away from a starting point

1D 

displacement could be positive or negative

Distance traveled: $|s|$ total movement occured

Example: 400m running track: 1 lap: displacement $= 0$m
distance: 400m



Velocity vs. Speed

Velocity: rate of change of displacement $V = \frac{ds}{dt}$ (meters) / (time)

$V>0$ "moving forward", $V<0$ "moving backward", $V=0$ stationary (at rest)

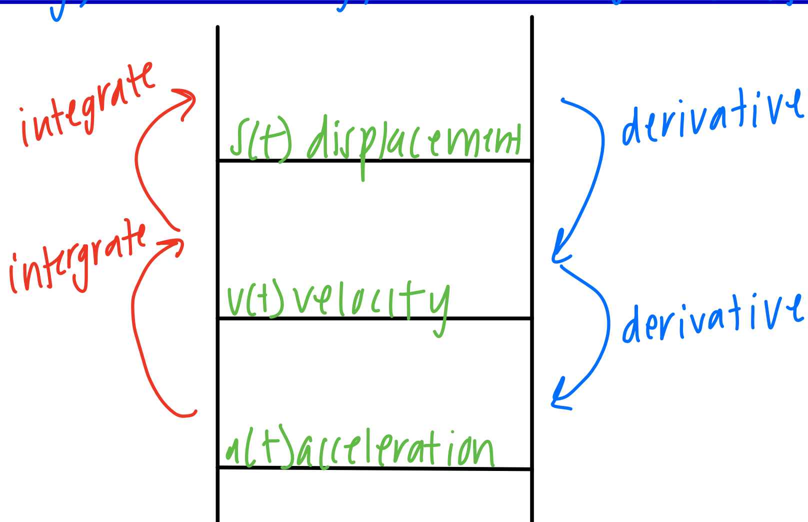
Speed: $|v|$ always positive

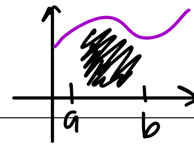
Acceleration

Acceleration: rate of change of velocity $a = \frac{dv}{dt} = \frac{d^2s}{dt^2}$

$a>0$ increasing velocity, $a<0$ decreasing velocity, $a=0$ constant velocity (not moving or constant speed)

Flow-chart:





distance + displacement same

Using integration to find displacement and distance traveled

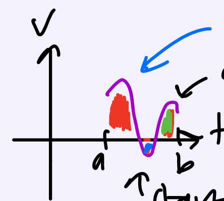
If $v(t)$ is the velocity function for an object,

* $\int_a^b v(t)\ dt$ gives the displacement from $t=a$ to $t=b$.

* Distance traveled between times $t=a$ and $t=b$ is the total area under a velocity curve.

In other words,

total distance traveled $= \int_a^b |v|\, dt$

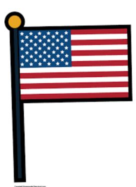


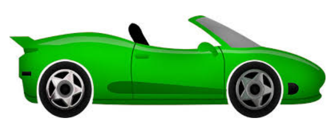
Total distance traveled from t_1 to t_2

$$\text{distance} = \int_{t_1}^{t_2} |v(t)|\, dt$$

Example: The velocity of a car t seconds after passing a flag is modeled by $v = 17 - 4t$ for $0 \le t \le 5$.

might go backward





a) What is the initial speed?

$t = 0$

$v = 17 - 4(0) = 17\ ms^{-1}$ m/s

b) Find the acceleration of the car? $a = \frac{dv}{dt}$

$a = -4ms^{-2}$

4.25 (time)

c) What is the maximum displacement from the flag? does it stop? $v = 0$ $17 - 4t = 0$

$\frac{17}{4} = t < 5$ yes.

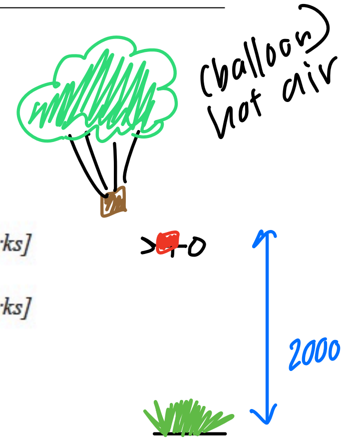
(calc)

$s = \int_0^{4.25} 17 - 4t\ dt = 36.125\ m$

d) Find the distance the car travels.

$d_1 = \int_{4.25}^{5} 17 - 4t\, dt = -1.125$

$d = 36.125 + 1.125 = 37.25$



math
8

3. *[Maximum mark: 6]*

A skydiver jumps from a stationary balloon at a height of 2000 m above the ground. Her velocity, v m s^{-1}, t seconds after jumping, is given by $v = 50(1 - e^{-0.2t})$.

(a) Find her acceleration 10 seconds after jumping. *[3 marks]*

(b) How far above the ground is she 10 seconds after jumping? *[3 marks]*

X = X graph

a) $a = \frac{dv}{dt}(10) = 1.35\ ms^{-2}$ M1 M1 M1

b) $s = \int_0^{10} 50(1 - e^{-0.2t})\,dt = 283.8\text{ m}$

$2000 - 283.8 = 1716.2\text{ m}$

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6. *[Maximum mark: 7]*

The acceleration, a m s^{-2}, of a particle at time t seconds is given by

$$a = \frac{1}{t} + 3\sin 2t, \text{ for } t \geq 1.$$

The particle is at rest when $t = 1$. V=0 when t=1

Find the velocity of the particle when $t = 5$.

$v = \int a\,dt = \int \frac{1}{t} + 3\sin 2t$

$v = \ln t - \frac{3}{2}\cos 2t + c$

$0 = \ln 1 - \frac{3}{2}\cos 2(1) + c$

$-0.624 = c$

$v = \ln 5 - \frac{3}{2}\cos(2\cdot 5) - 0.624$

$v = 2.24\ ms^{-1}$

May 2009 Paper 1

11. *[Maximum mark: 17]*

In this question s represents displacement in metres and t represents time in seconds.

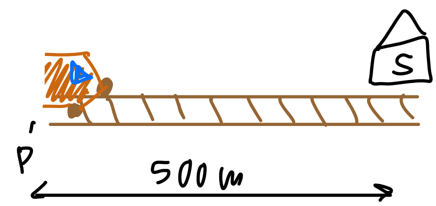
not acceleration!

The velocity v m s^{-1} of a moving body is given by $v = 40 - at$ where a is a non-zero constant.

(a) (i) If $s = 100$ when $t = 0$, find an expression for s in terms of a and t.

(ii) If $s = 0$ when $t = 0$, write down an expression for s in terms of a and t. *[6 marks]*

Trains approaching a station start to slow down when they pass a point P. As a train slows down, its velocity is given by $v = 40 - at$, where $t = 0$ at P. The station is 500 m from P.



(b) A train M slows down so that it comes to a stop at the station.

(i) Find the time it takes train M to come to a stop, giving your answer in terms of a.

$v = 0$

(ii) Hence show that $a = \frac{8}{5}$. *[6 marks]*

(c) For a different train N, the value of a is 4.
Show that this train will stop **before** it reaches the station. *[5 marks]*

a) i) $s = \int 40 - at\, dt$

$s = 40t - \frac{1}{2}at^2 + C$

$100 = C$

$s = 40t - \frac{1}{2}at^2 + 100$

↑did not use bc train was not 100 m away from p

ii) $0 = C$

$s = 40t - \frac{1}{2}at^2$

use equation

b) i) $v = 40 - at = 0 \rightarrow t = \frac{40}{a}$

ii) $500 = 40\left(\frac{40}{a}\right) - \frac{1}{2}a\left(\frac{40}{a}\right)^2$

$500 = \frac{1600}{a} - \frac{1}{2}\cancel{a}\frac{1600}{\cancel{a}^2}$

$500 = \frac{1600}{a} - \frac{800}{a}$

$500 = \frac{800}{a}$ $500a = 800$ $a = \frac{8}{5}$

c) Train 2: $v = 40 - 4t$

METHOD #1

$v = 0 = 40 - 4t$

$t = 10$

$s = 40(10) - \frac{1}{2}(4)(10)^2$

$s = 400 - 200 = 200m$

$200 < 500$

$\therefore$ stops before station

METHOD #2

$v = 40 - 4t$

$a' = v' = -4$

slowing down faster → so won't reach station bc Train 1 does.

Train 1

$a = -\frac{8}{5}$