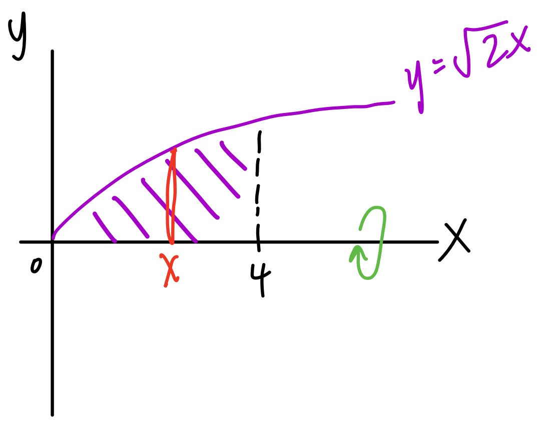
NOT ON IB EXAM!!!

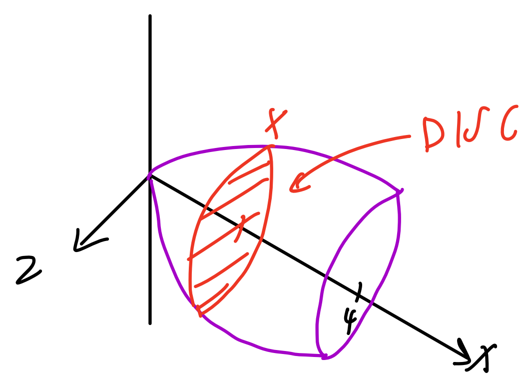
 paraboloid: 3-D parabola

8.4) Volumes of Revolution: DISCS

- Similar to how we used ___rectangles___, and hence an ___integral___, to approximate an **area**, we can use a similar practice to compute VOLUMES.
- Decreasing the widths ___Δx___ of the ___DISCS___ will lead to a more ___accurate___ approximation.
- If we shrink ___$\Delta x \to 0$___ the ___width___ become so thin our sum becomes an ___integral___.

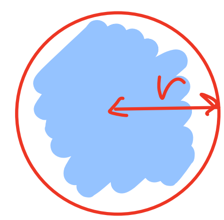
Example: Find the volume of the solid formed when the graph of $y = \sqrt{2x}$ over [0,4] is rotated about the x-axis through an angle of 2π.





Each cross-section here is a ___disc/cylinder___.

$\text{Area} = \pi r^2 = \pi y^2$



The radius of the disc changes depending on ___x___...

width $= \Delta x$

but we know ___$r = y = \sqrt{2x}$___.

$\text{Area} = \pi(\sqrt{2x})^2 = 2\pi x$

$\text{Volume} = 2\pi x \cdot \text{width} = 2\pi x \Delta x$

For the full solid we just sum all the discs between $x = 0$ and $x = 4$.

shrink $\Delta x \to 0$

$V = \int_0^4 2\pi x \, dx = \pi \int_0^4 2x \, dx = 16\pi u^3$

units!

Summary:

- If a region bounded by a closed interval [a,b] on the x-axis, the volume of the Solid of revolution is:

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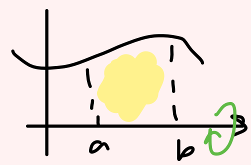
Volume of revolution (rotation)

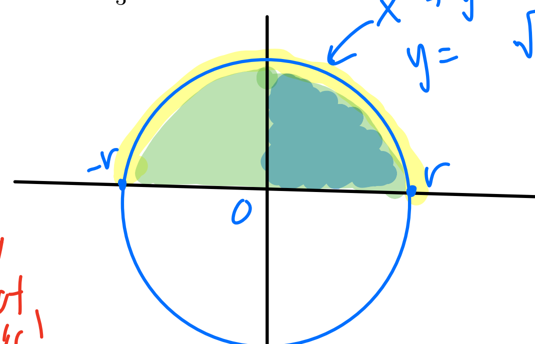
$$V = \int_a^b \pi y^2 \quad dx = \pi \int_a^b y^2 dx$$



Example: Prove that the volume of a sphere with radius r is $V = \frac{4}{3}\pi r^3$.

$x^2+y^2=r^2$

$y=\sqrt{r^2-x^2}$



$$V=\pi\int_{-r}^{r}\left(\sqrt{r^2-x^2}\right)^2 dx$$

$$V=2\pi\int_0^r r^2-x^2\,dx$$

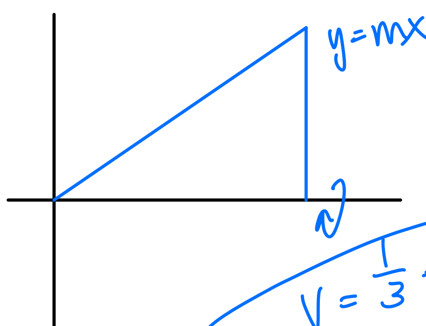
constant!!! bc you're integrating x's not r's!

$$V=2\pi\left[r^2x-\frac{x^3}{3}\right]_0^r$$

$$=2\pi\left[\left(r^3-\frac{r^3}{3}\right)-(0-0)\right]=2\pi\left(\frac{3r^3}{3}-\frac{r^3}{3}\right)$$

$$=2\pi\left(\frac{2r^3}{3}\right)$$

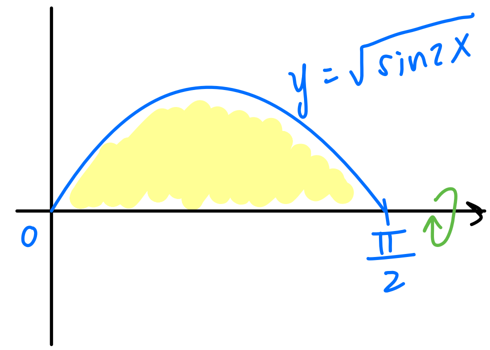
$$=\frac{4}{3}\pi r^3$$



$$V=\frac{1}{3}\pi r^2 h$$

Example: Find the volume of the solid generated when the region enclosed by $y = \sqrt{\sin 2x}$, $x = 0$ and $x = \pi/2$ is rotated about the x-axis $360°$.

$$V = \pi \int_a^b y^2 \, dx$$



$$V = \pi \int_0^{\frac{\pi}{2}} \left(\sqrt{\sin 2x}\right)^2 dx$$

$$V = \pi \int_0^{\frac{\pi}{2}} \sin 2x \, dx$$

$$V = \pi \left[-\frac{1}{2}\cos 2x\right]_0^{\frac{\pi}{2}} = \pi\left(-\frac{1}{2}\underbrace{\cos \pi}_{-1} - -\frac{1}{2}\underbrace{\cos 0}_{1}\right)$$

$$= \pi\left(\frac{1}{2} + \frac{1}{2}\right) = \pi u^3$$